Quantifying progress towards fusion energy gain: the Lawson criterion

2024 PPPL / SULI Introduction to Fusion Energy and Plasma Physics Course

Sam Wurzel
sam@fusionenergybase.com

Fusion Energy Base

June 21st, 2024

Backstory

University of Colorado **Boulder**

2005

Octopart

2007

2015

Altium

2015

Fusion Energy Base

2019

arpa·e

2022

Fusion Energy Base

2024

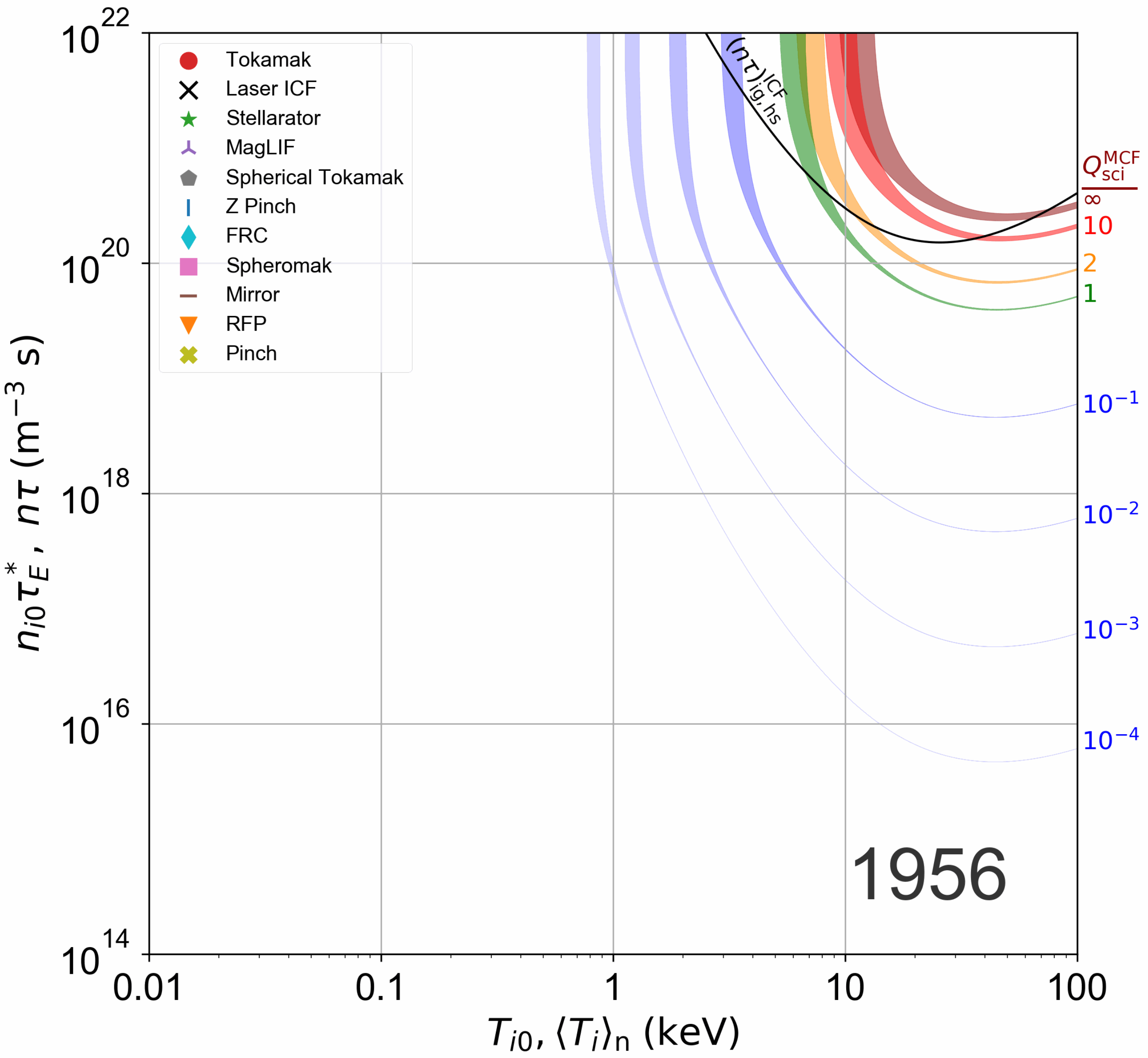
Outline

- Punchline first: progress towards fusion energy breakeven and gain
- Review of the Lawson criterion following Lawson's 1955 approach
- Extend Lawson's analysis to steady-state MCF and pulsed ICF
- Advanced fusion fuels

PROGRESS TOWARDS FUSION BREAKEVEN AND GAIN

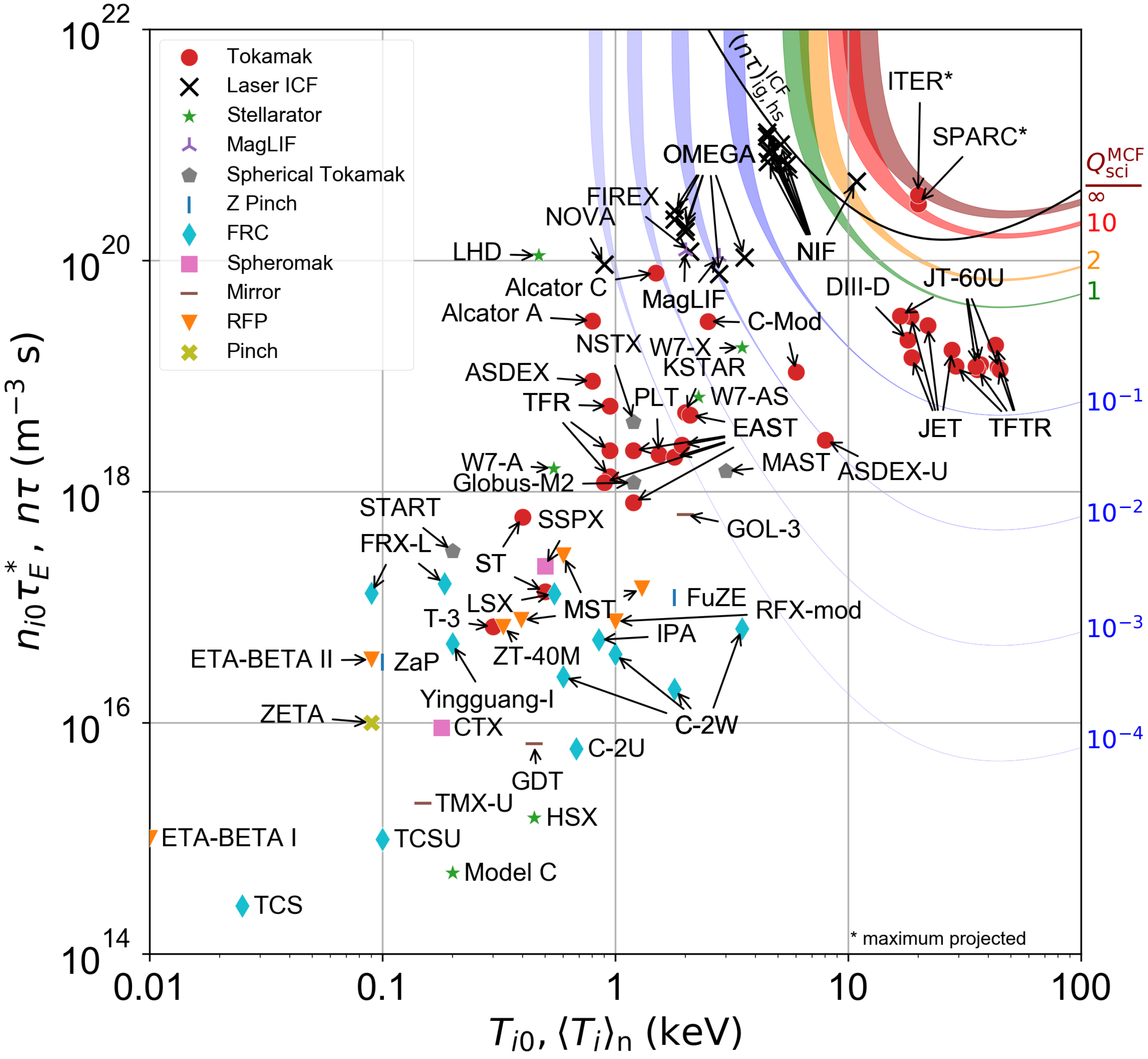
Progress towards energy gain

[Animation]



Adapted from S.E. Wurzel and S. C Hsu
Physics of Plasmas **29**, 062103 (2022)

Progress towards energy gain and ignition



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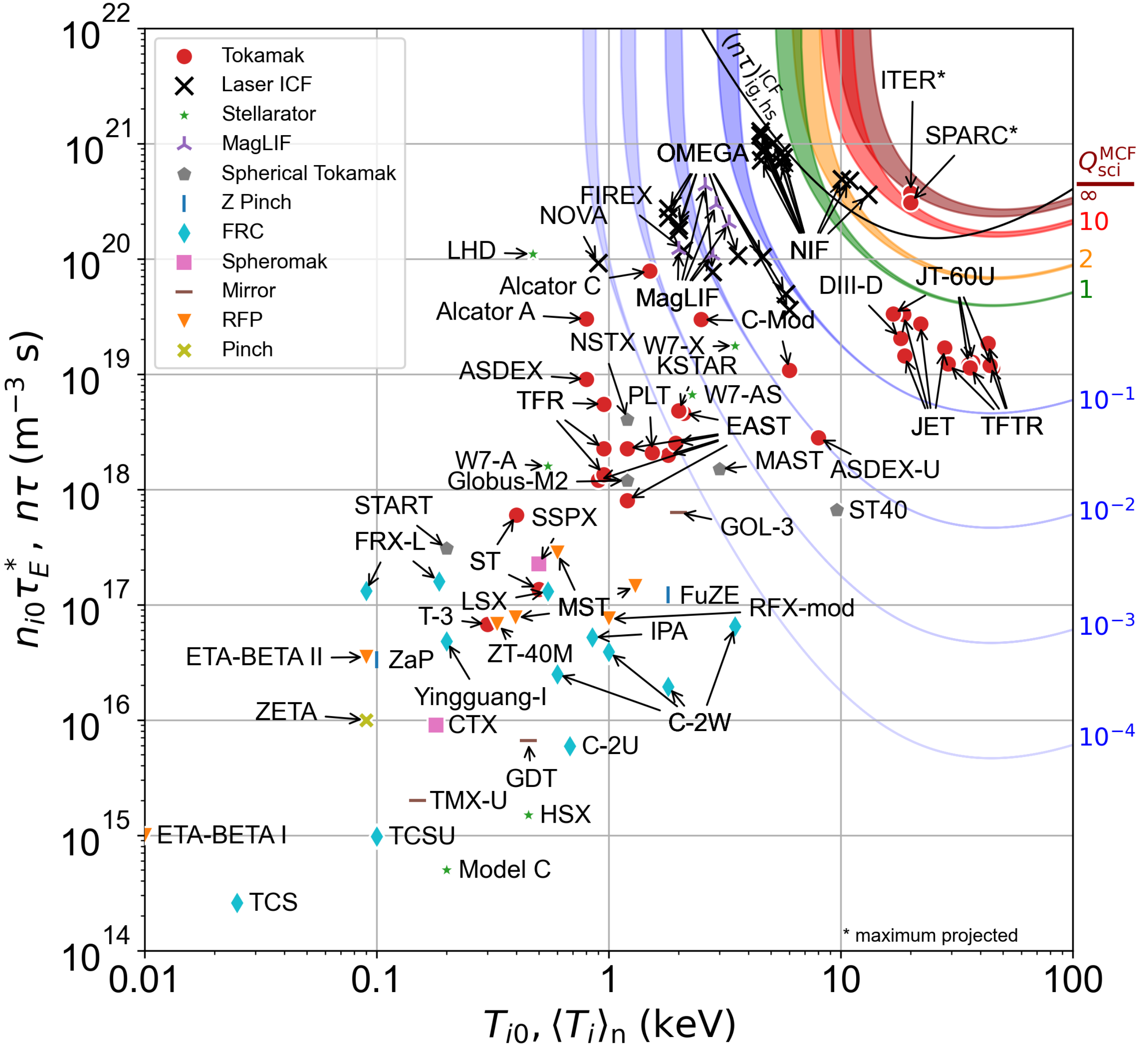
Progress towards energy gain and ignition



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LAWSON’S 1955 PAPER

“Some criteria for a useful thermonuclear reactor” Lawson (1955)

A.E.R.E. GP/R 1807

2 EXTRA COPIES

A.E.R.E. GP/R 1807

COPY 1

Re-graded from DC 5 d-1. February 11th 1957.

ATOMIC ENERGY
RESEARCH ESTABLISHMENT

SOME CRITERIA FOR A USEFUL
THERMONUCLEAR REACTOR

by

J. D. LAWSON

HARWELL, BERKS.
1955

INTRODUCTION

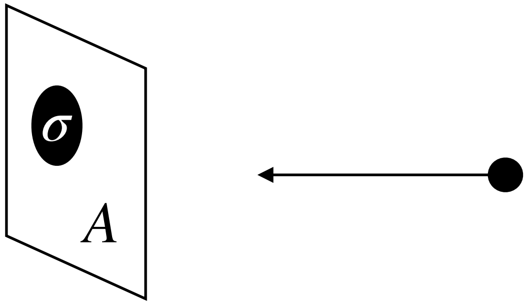
In this report the power balance in thermonuclear reactors is considered and criteria which must be satisfied in a useful reactor are found.

...

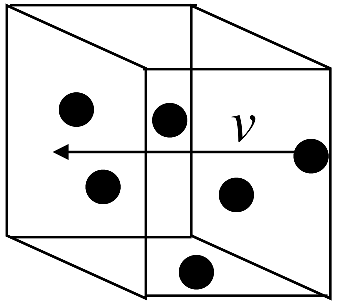
Various idealized systems will now be analysed. Possible methods of setting up such systems will not however be discussed.

J. D. Lawson, “Some criteria for a useful thermonuclear reactor,” “Technical Report No. GP/R 1807 (1955).

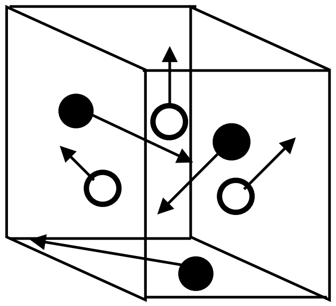
Fusion cross section σ and thermal reactivity $\langle \sigma v \rangle$



σ / A is a probability of a "hit"



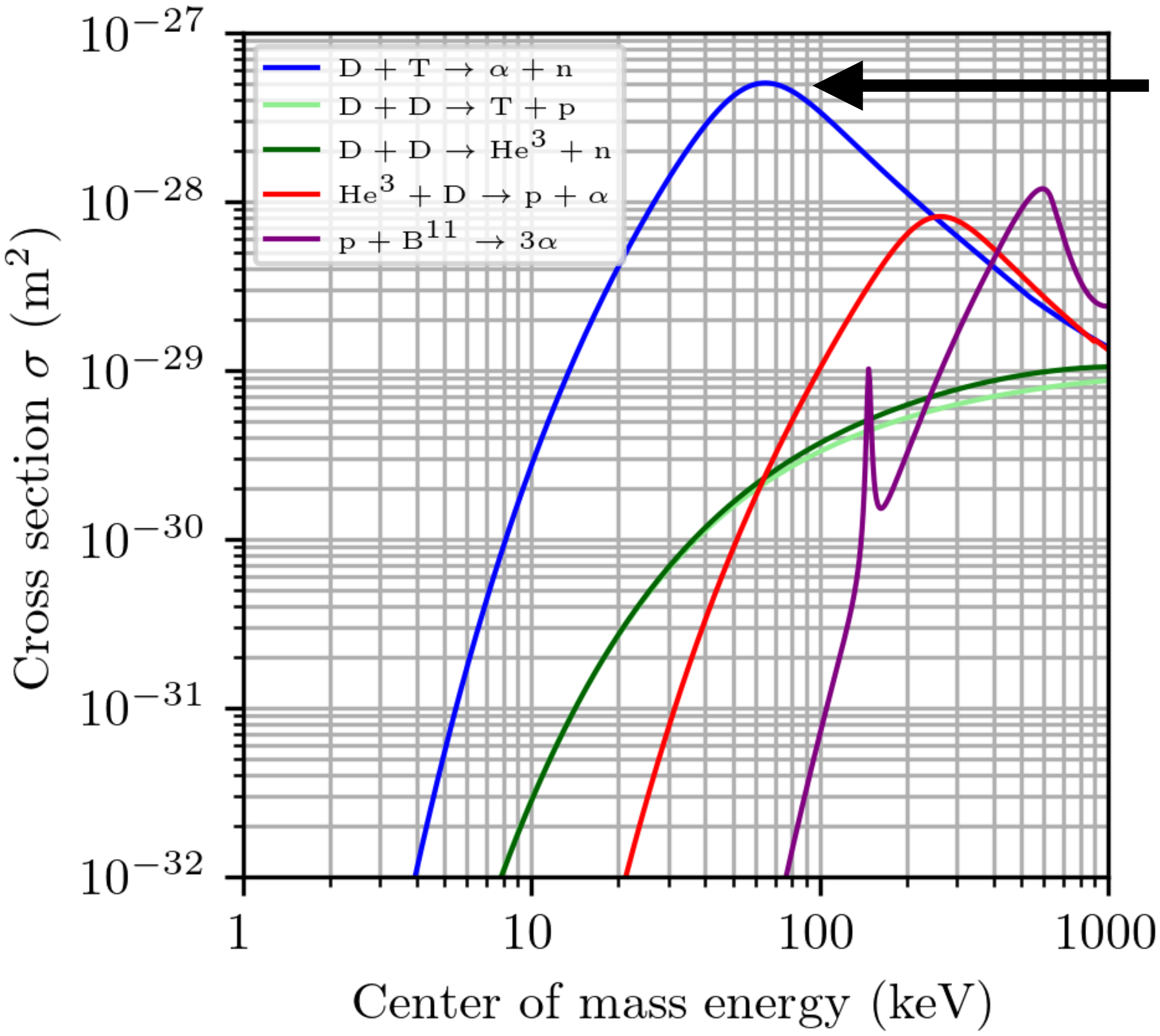
$\sigma v n$ is the rate of "hits" on a stationary target of density n by incoming particle with velocity v



$n_1 n_2 \langle \sigma v \rangle V$ is the rate of "hits" between particles of density n_1 and n_2 with Maxwellian velocity distribution in volume V.

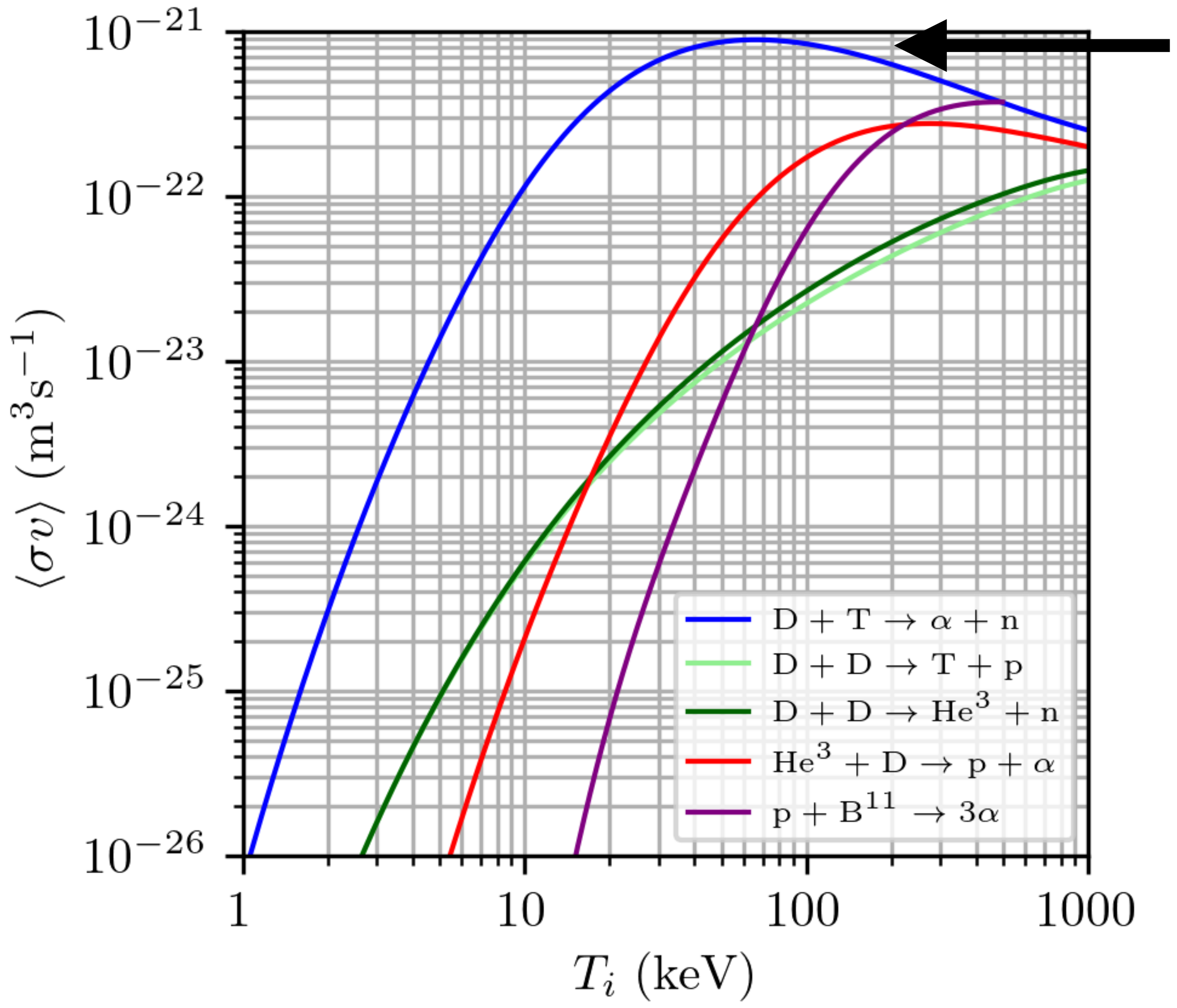
$\langle \sigma v \rangle$ is the cross section times the relative velocity averaged over a Maxwellian velocity distribution and is a function of temperature T.

Fusion cross sections



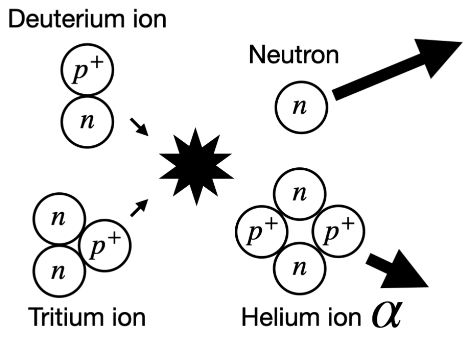
D-T reaction has highest cross section at lowest CM energy

Fusion thermonuclear reactivities and fusion power



D-T Fusion has the highest reactivity at the lowest temperature

$$D + T \rightarrow \alpha \ (3.5 \text{ MeV}) + n \ (14.1 \text{ MeV})$$

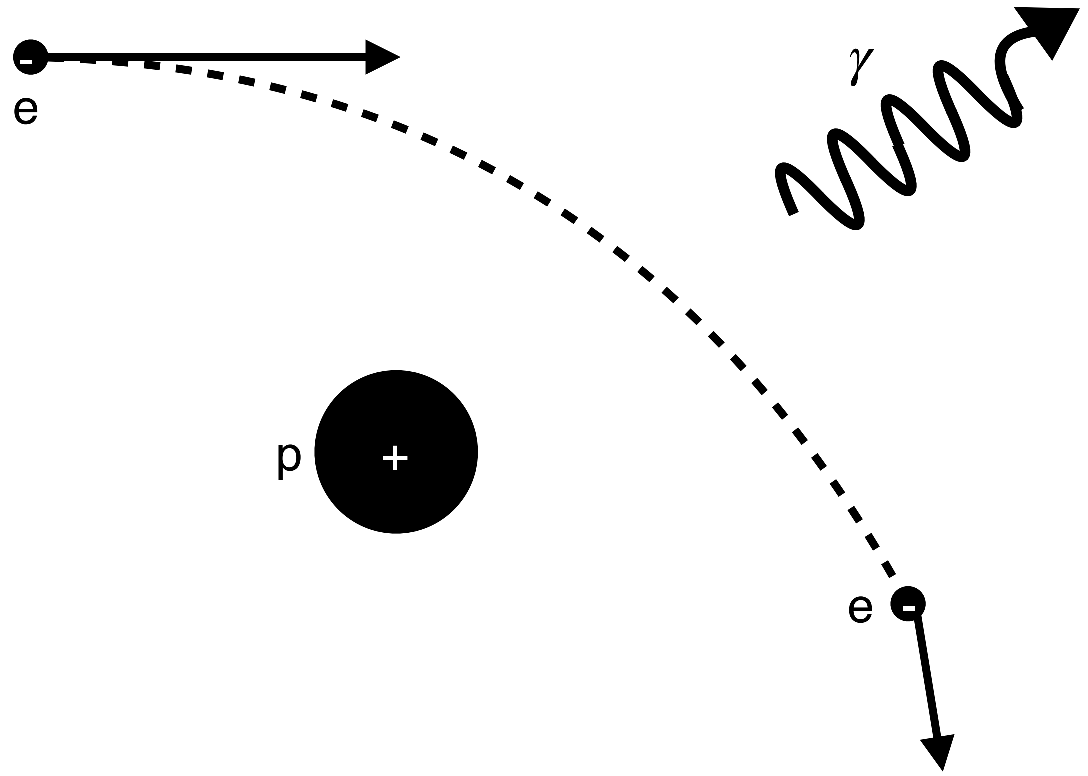


Power produced by fusions in a 50/50 deuterium-tritium plasma of volume V:

$$P_F = n_D n_T \langle \sigma v \rangle \epsilon_F V$$

ϵ_F is the total energy per fusion (17.6 MeV)

Bremsstrahlung in a hydrogen plasma



Power emitted as bremsstrahlung
In a hydrogen plasma:

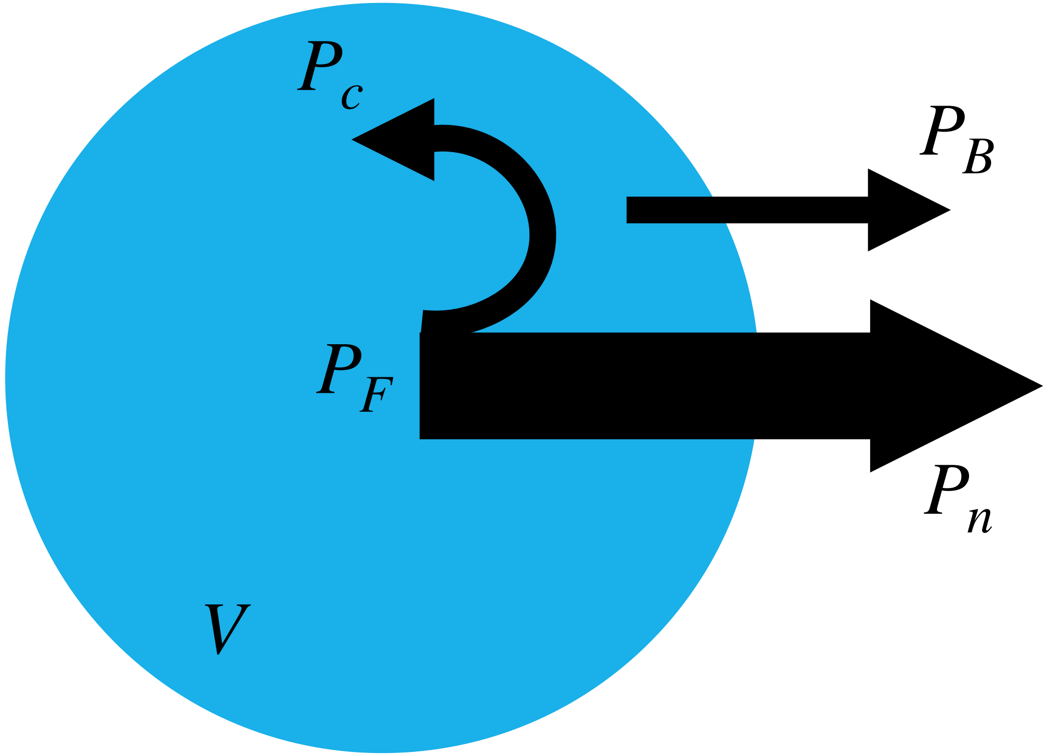
$$P_B = C_B n^2 T^{1/2} V$$

C_B is a constant.

Lawson's first scenario: steady state

- Heating power from charged fusion products must equal or exceed bremsstrahlung power

$n_D = n_T$ (50% deuterium, 50% tritium)
$n = n_D + n_T$ (pure hydrogen plasma)
$T = T_i = T_e$ (thermal equilibrium)
Perfect confinement
Charged fusion products self-heat



Bremsstrahlung power

$$P_B = C_B n^2 T^{1/2} V$$

Fusion power of alphas

$$P_c = f_c P_F = f_c \frac{1}{4} n^2 \langle \sigma v \rangle \epsilon_F V$$

f_c is the fraction of energy in charged fusion products (20% for D-T)

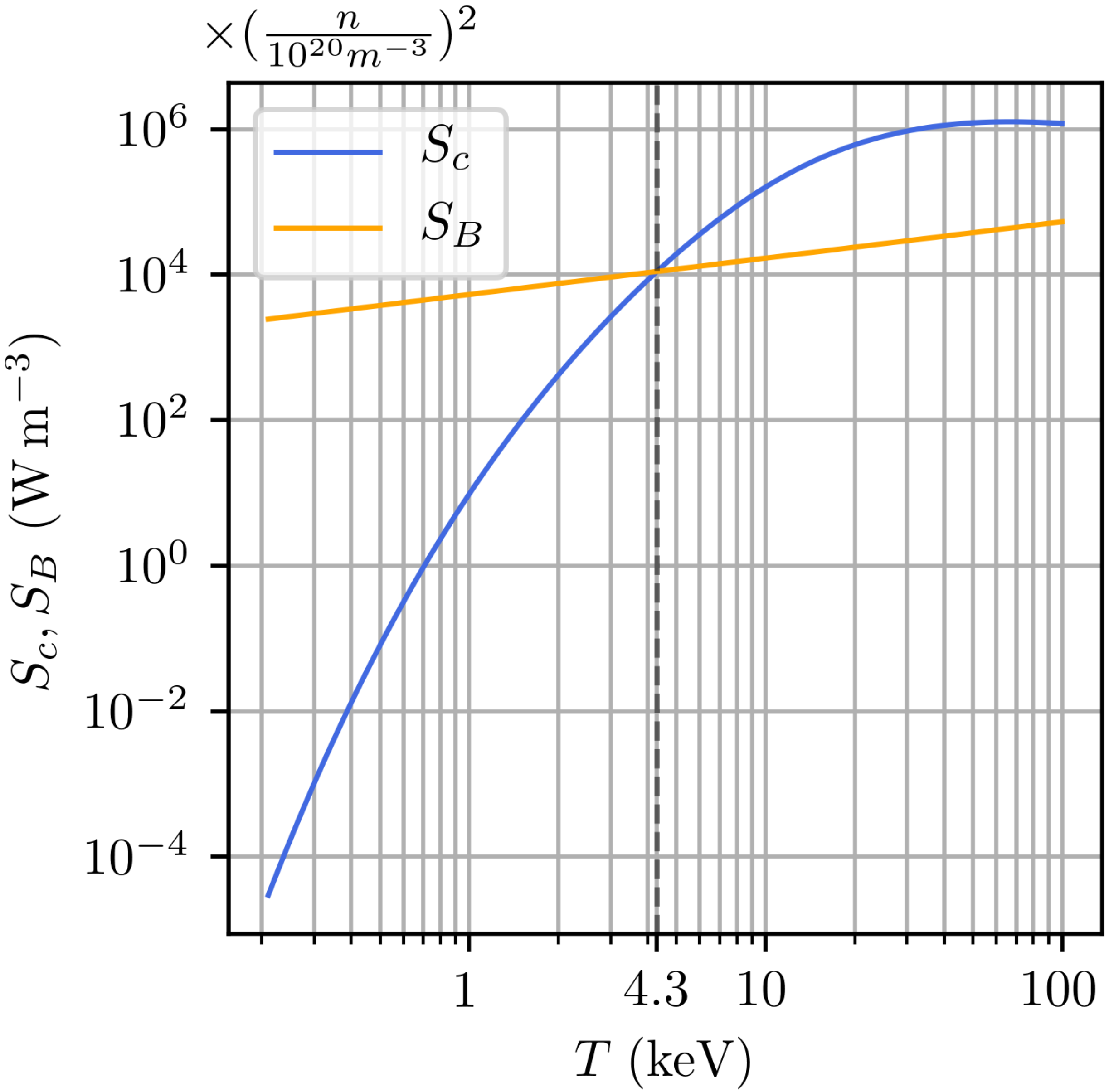
Ideal ignition temperature

Charged fusion power equals bremsstrahlung power at $T = 4.3$ keV, when

$$f_c \frac{1}{4} n^2 \langle \sigma v \rangle \epsilon_F V = C_B n^2 T^{1/2} V,$$

independent of density.

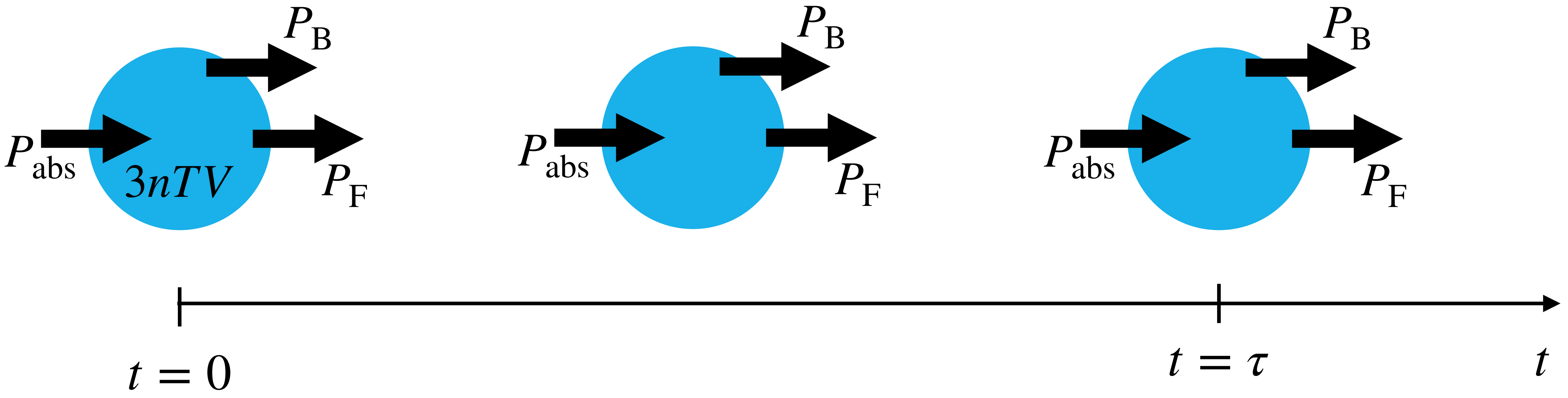
$S_c = P_c / V$, $S_B = P_B / V$



Lawson's second scenario: pulsed

- Plasma temperature **instantaneously** raised from zero to temperature T at $t = 0$
- Absorbed external heating power P_{abs} applied over pulse duration τ

$n_D = n_T$ (50% deuterium, 50% tritium)
$n = n_D + n_T$ (pure hydrogen plasma)
$T = T_i = T_e$ (thermal equilibrium)
Perfect confinement
All fusion products exit the plasma (no self heating)



Q_{fuel}

$$Q_{\text{fuel}} = \frac{\text{Fusion energy}}{\text{Heating energy absorbed by fuel}}$$

(Lawson used R)

Emergence of the Lawson parameter $n\tau$

$$Q_{\text{fuel}} = \frac{\tau P_F}{\tau P_{\text{abs}} + 3nTV} = \frac{\tau P_F}{\tau P_B + 3nTV}$$

$$= \frac{P_F/(3n^2TV)}{P_B/(3n^2TV) + 1/n\tau} = \frac{\langle\sigma v\rangle\epsilon_F/12T}{C_B/3T^{1/2} + \boxed{1/n\tau}}$$

Q_{fuel} is a function of temperature T and “Lawson parameter” $n\tau$.

Lawson's requirement for a "useful" system

Surplus power condition

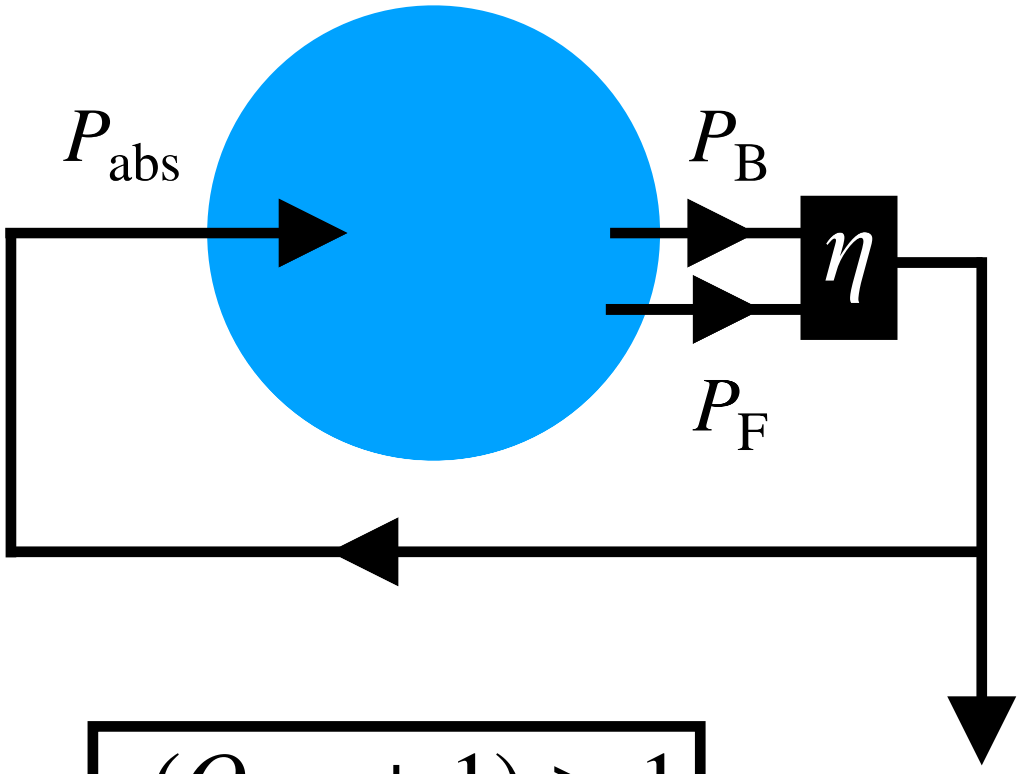
$$(P_B + P_F)\eta > P_{abs}$$

Steady state condition

$$P_B = P_{abs}$$

Steady state gain

$$Q_{fuel} = \frac{P_F}{P_{abs}}$$



$$\eta(Q_{fuel} + 1) > 1$$

Lawson assumed $\eta \approx 1/3$, requiring $Q_{fuel} > 2$.

$Q_{\text{fuel}} > 2$ requires high threshold of T and $n\tau$

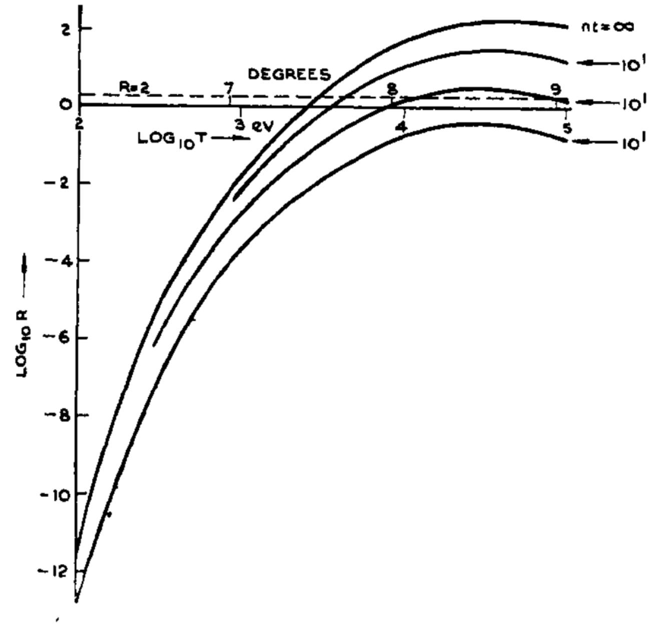
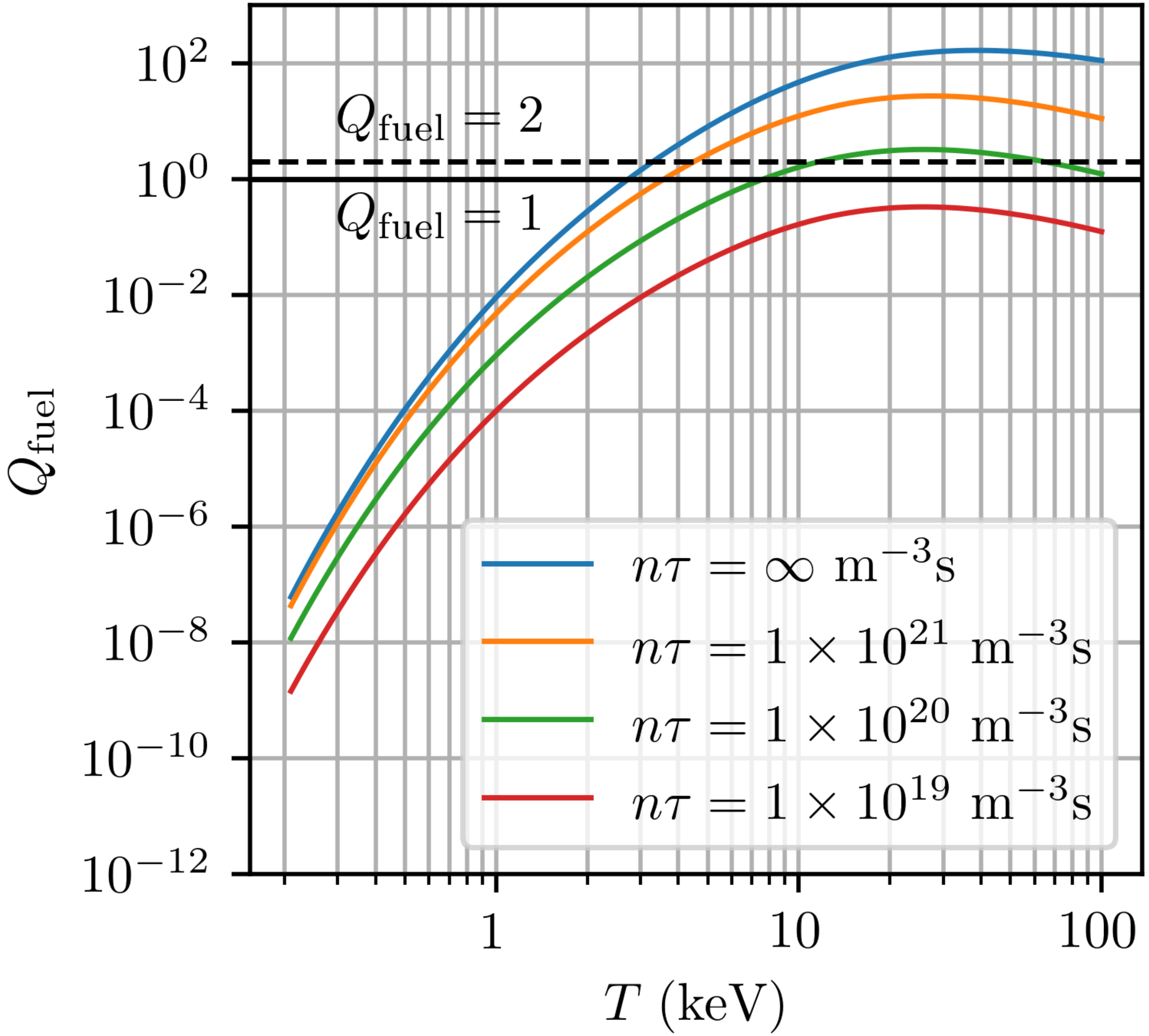


Figure 2. Variation of R with T for various values of nt fo



Lawson's conclusion

CONCLUSION

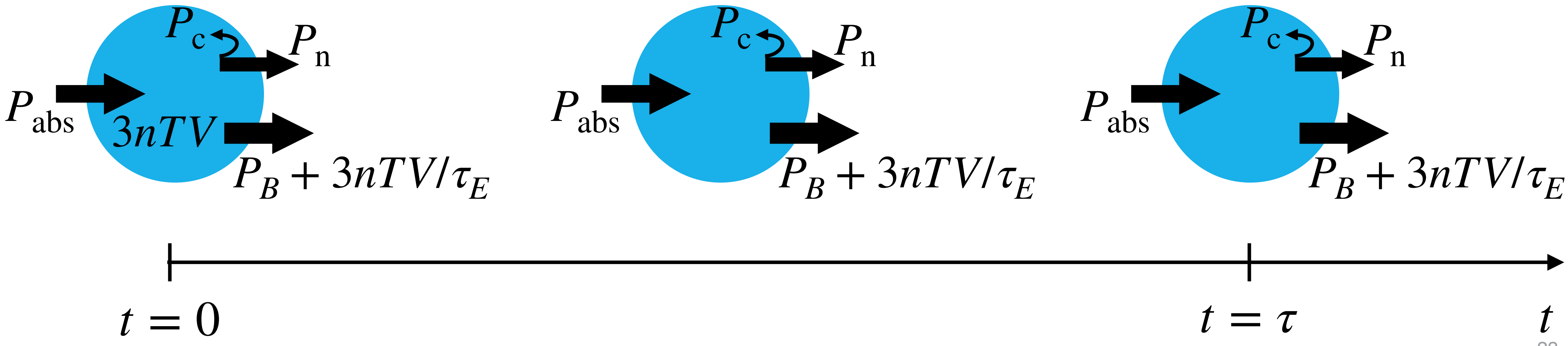
Even with the most optimistic possible assumptions it is evident that the conditions for the operation of a useful thermonuclear reactor are very severe.

EXTENDING LAWSON'S ANALYSIS

Extending Lawson's analysis to include thermal conduction and self heating

- Plasma temperature **instantaneously** raised from zero to temperature T at $t = 0$ and maintained at T until $t = \tau$
- Thermal-conduction power loss: $3nTV/\tau_E$
- Absorbed external heating power P_{abs} and self heating P_c applied over pulse duration τ

$n_D = n_T$ (50% deuterium, 50% tritium)
$n = n_D + n_T$ (pure hydrogen plasma)
$T = T_i = T_e$ (thermal equilibrium)
Imperfect confinement: τ_E is finite



Lawson-type analysis

$$Q_{\text{fuel}} = \frac{\tau P_F}{3nTV + \tau P_{\text{abs}}} \qquad P_{\text{abs}} + P_c = P_B + 3nTV/\tau_E$$

$$Q_{\text{fuel}} = \frac{\langle \sigma v \rangle \epsilon_F / 12T}{C_B/3T^{1/2} - f_c \langle \sigma v \rangle \epsilon_F / 12T + \boxed{1/n\tau + 1/n\tau_E}}$$

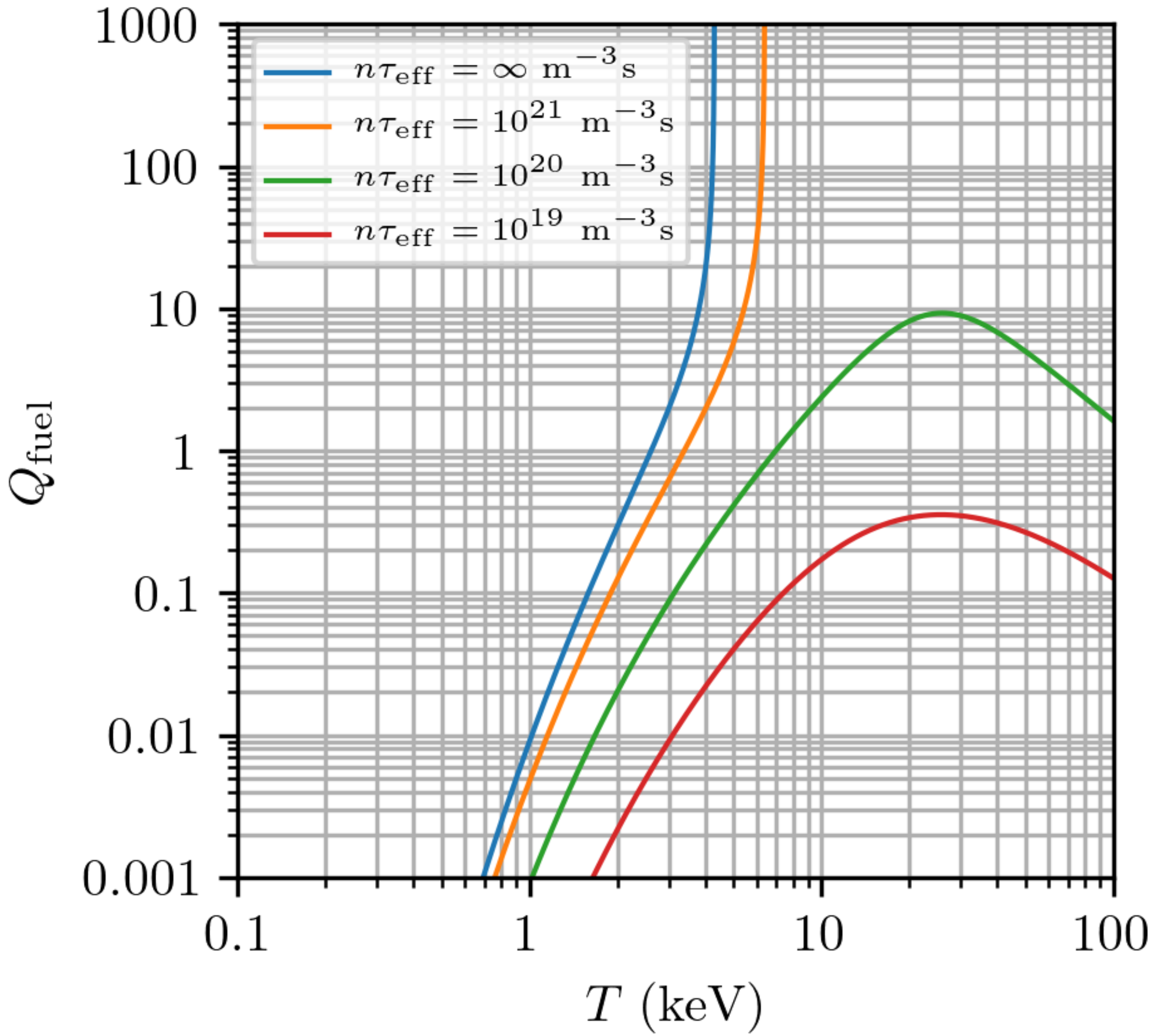
$$n\tau_{\text{eff}} = \frac{3T}{(f_c + Q_{\text{fuel}}^{-1})\langle \sigma v \rangle \epsilon_F / 4 - C_B T^{1/2}}$$

$$\boxed{\tau_{\text{eff}} = \frac{\tau \tau_E}{\tau + \tau_E}}$$

Characteristic times add like resistors in parallel

- If $\tau \ll \tau_E$ Lawson parameter is $n\tau$ and ICF-like
- If $\tau_E \ll \tau$ Lawson parameter is $n\tau_E$ and MCF-like
- If $\tau_E \sim \tau$ both must be considered

Q_{fuel} vs T for various values of $n\tau$



APPLICATION TO STEADY STATE MAGNETIC CONFINEMENT FUSION (MCF)

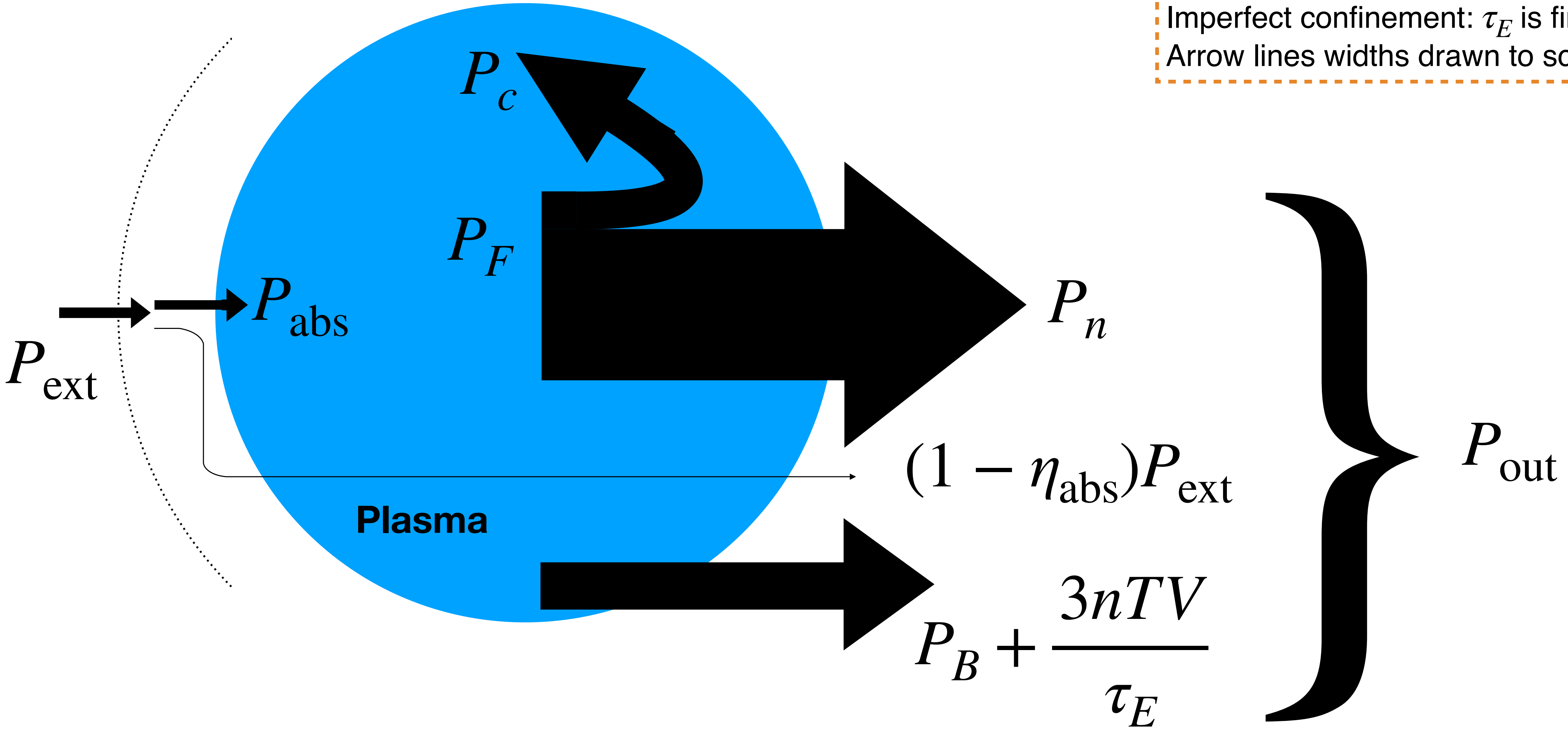
Q_{sci}

$$Q_{\text{sci}} = \frac{\text{Fusion power}}{\text{Heating power applied accross vacuum boundary}}$$

Limit of $\tau \to \infty, \tau_{\text{eff}} \to \tau_E$ describes idealized steady-state MCF

$Q_{\text{fuel}} = 20$ $\quad \eta_{\text{abs}} = 0.9$ $\quad Q_{\text{sci}} = \eta_{\text{abs}} Q_{\text{fuel}} = 18$

$n_D = n_T$ (50% deuterium, 50% tritium)
$n = n_D + n_T$ (pure hydrogen plasma)
$T = T_i = T_e$ (thermal equilibrium)
Imperfect confinement: τ_E is finite
Arrow lines widths drawn to scale



$Q_{\rm sci}$ and analysis of idealized steady-state MCF experiment

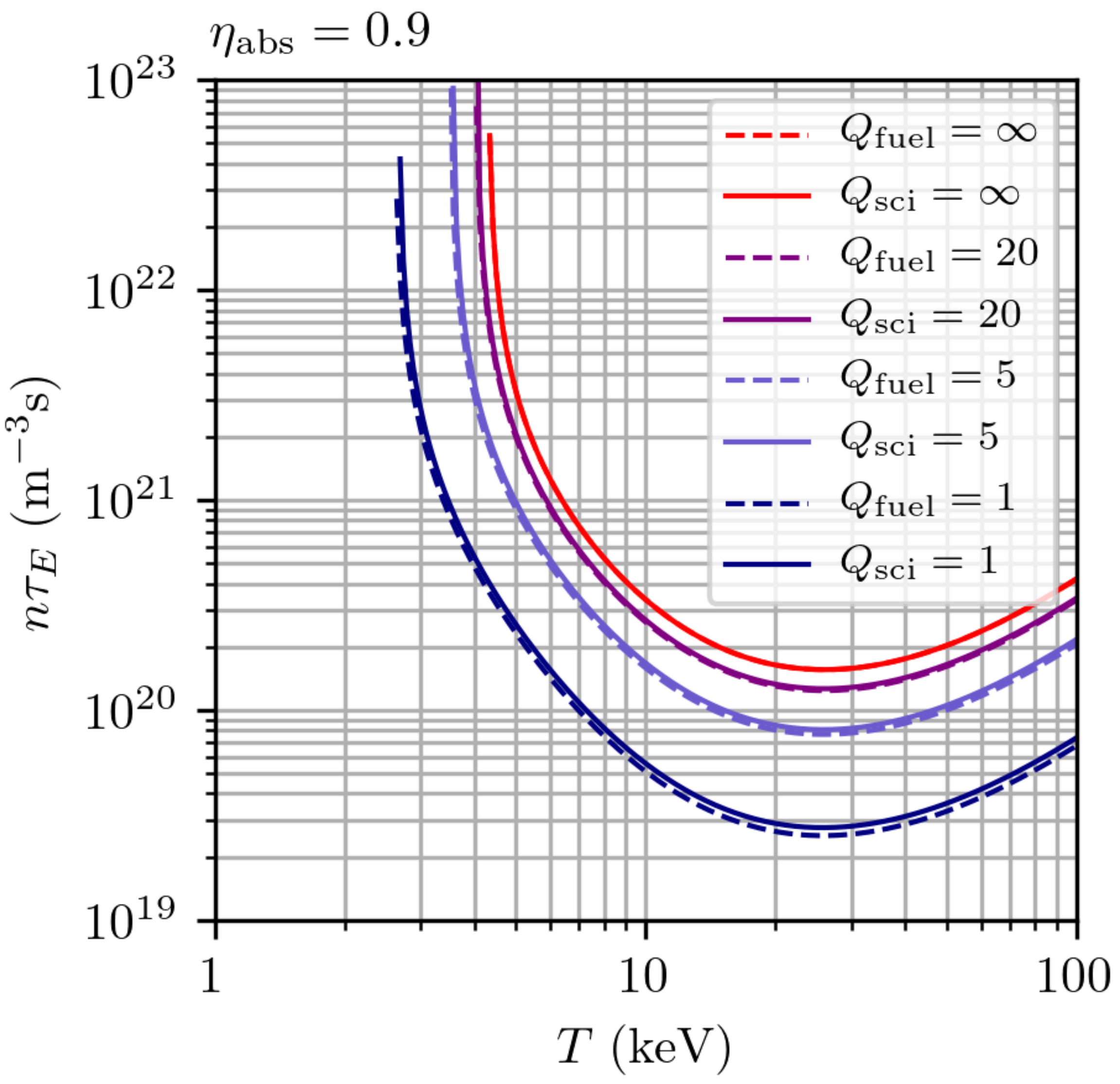
$$Q_{\rm sci} = \frac{P_F}{P_{\rm ext}} = \eta_{\rm abs} Q_{\rm fuel} < Q_{\rm fuel}$$

Power balance:

$$P_c + P_{\rm abs} = P_B + \frac{3nTV}{\tau_E}$$

$$n\tau_E = \frac{3T}{(f_c + Q_{\rm fuel}^{-1})\langle\sigma v\rangle\epsilon_F/4 - C_B T^{1/2}}$$

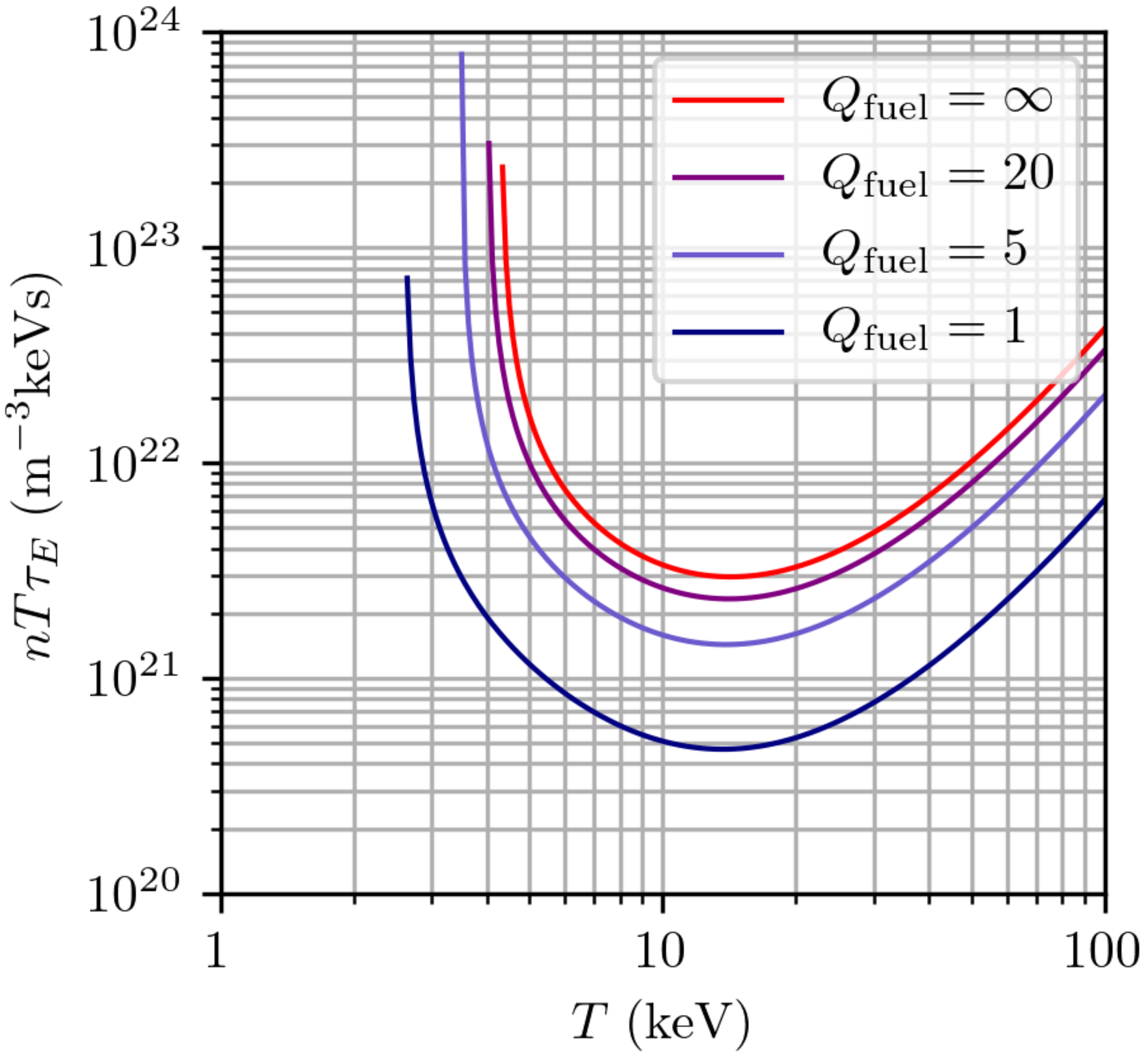
$$n\tau_E = \frac{3T}{(f_c + \eta_{\rm abs} Q_{\rm sci}^{-1})\langle\sigma v\rangle\epsilon_F/4 - C_B T^{1/2}}$$



Fusion "triple product"

$$nT\tau_E = \frac{1}{2} p\tau_E$$

$$nT\tau_E = \frac{3T^2}{(f_c + Q_{\mathrm{fuel}}^{-1})\langle\sigma v\rangle\epsilon_F/4 - C_B T^{1/2}}$$

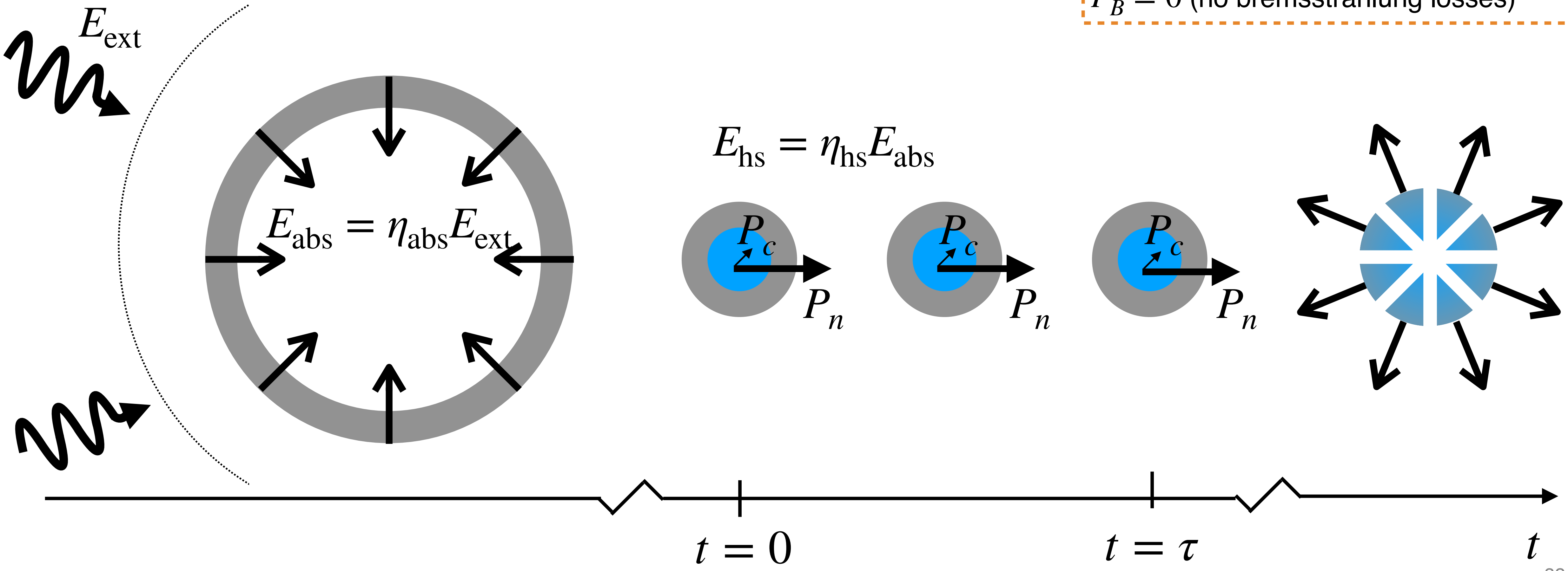


APPLICATION TO PULSED INERTIAL CONFINEMENT FUSION (ICF)

Limit of $\tau_E \to \infty$, $\tau_{\text{eff}} \to \tau$, and $P_B = 0$ describes idealized ICF

- Energy accounting over confinement duration τ of the hot-spot

$n_D = n_T$ (50% deuterium, 50% tritium)
$n = n_D + n_T$ (pure hydrogen plasma)
$T = T_i = T_e$ (thermal equilibrium)
$\tau_E = \infty$ (no thermal conduction losses)
$P_B = 0$ (no bremsstrahlung losses)



Q_{fuel} and analysis of idealized ICF hot-spot

$$Q_{\text{fuel}} = \frac{\tau P_F}{E_{\text{abs}}} = \frac{\tau P_F}{E_{\text{hs}}/\eta_{\text{hs}}}$$

Energy balance of hot-spot (low self heating)

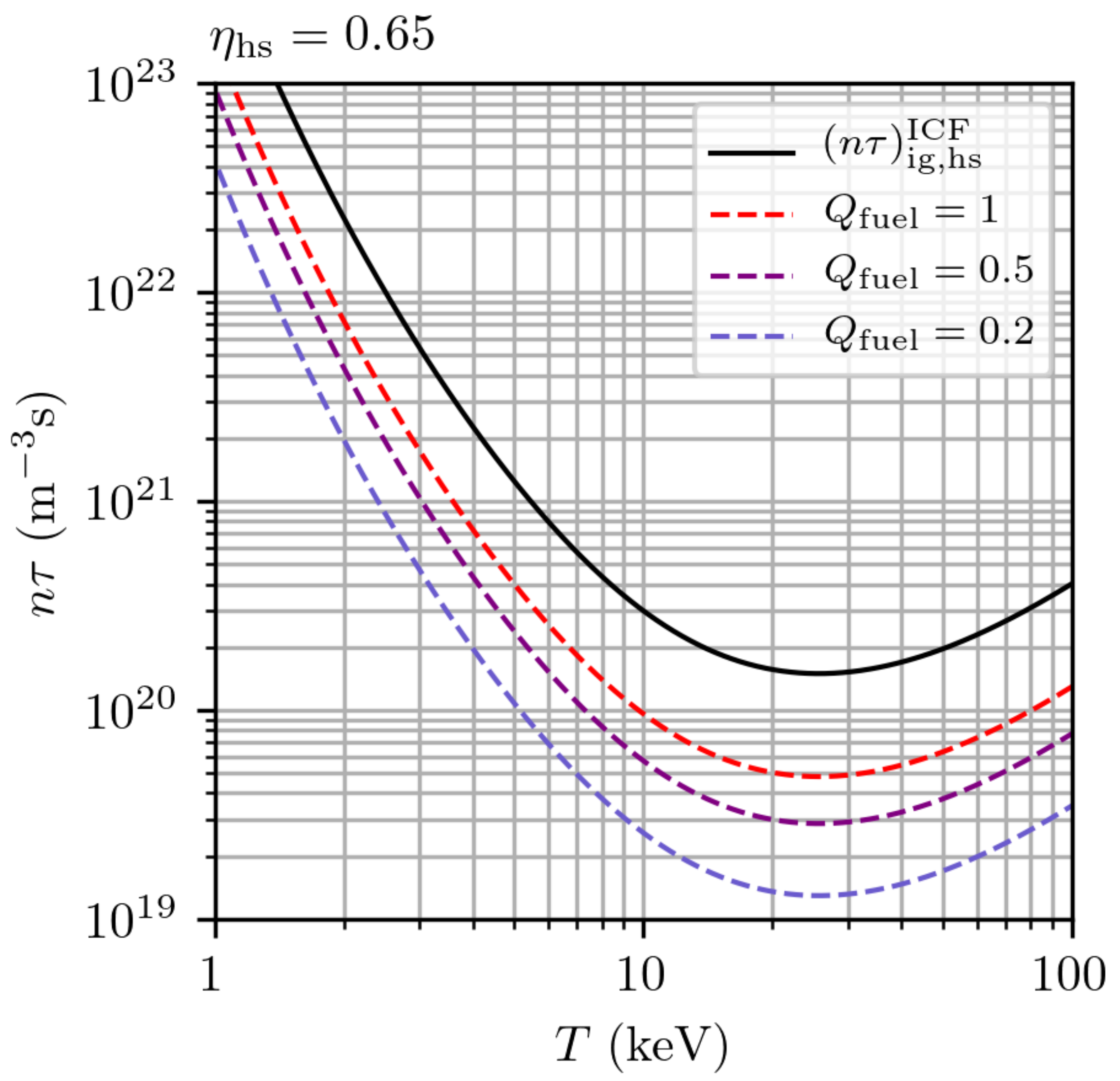
$$E_{\text{hs}} + \tau P_c = 3nTV$$

$$n\tau = \frac{12T}{(f_c + \eta_{\text{hs}} Q_{\text{fuel}}^{-1})\langle \sigma v \rangle \epsilon_F}$$

Self heating exceeds all losses (ignition)

$$\tau P_c = 3nTV$$

$$(n\tau)^{\text{ICF}}_{\text{ig,hs}} = \frac{12T}{\langle \sigma v \rangle \epsilon_\alpha}$$

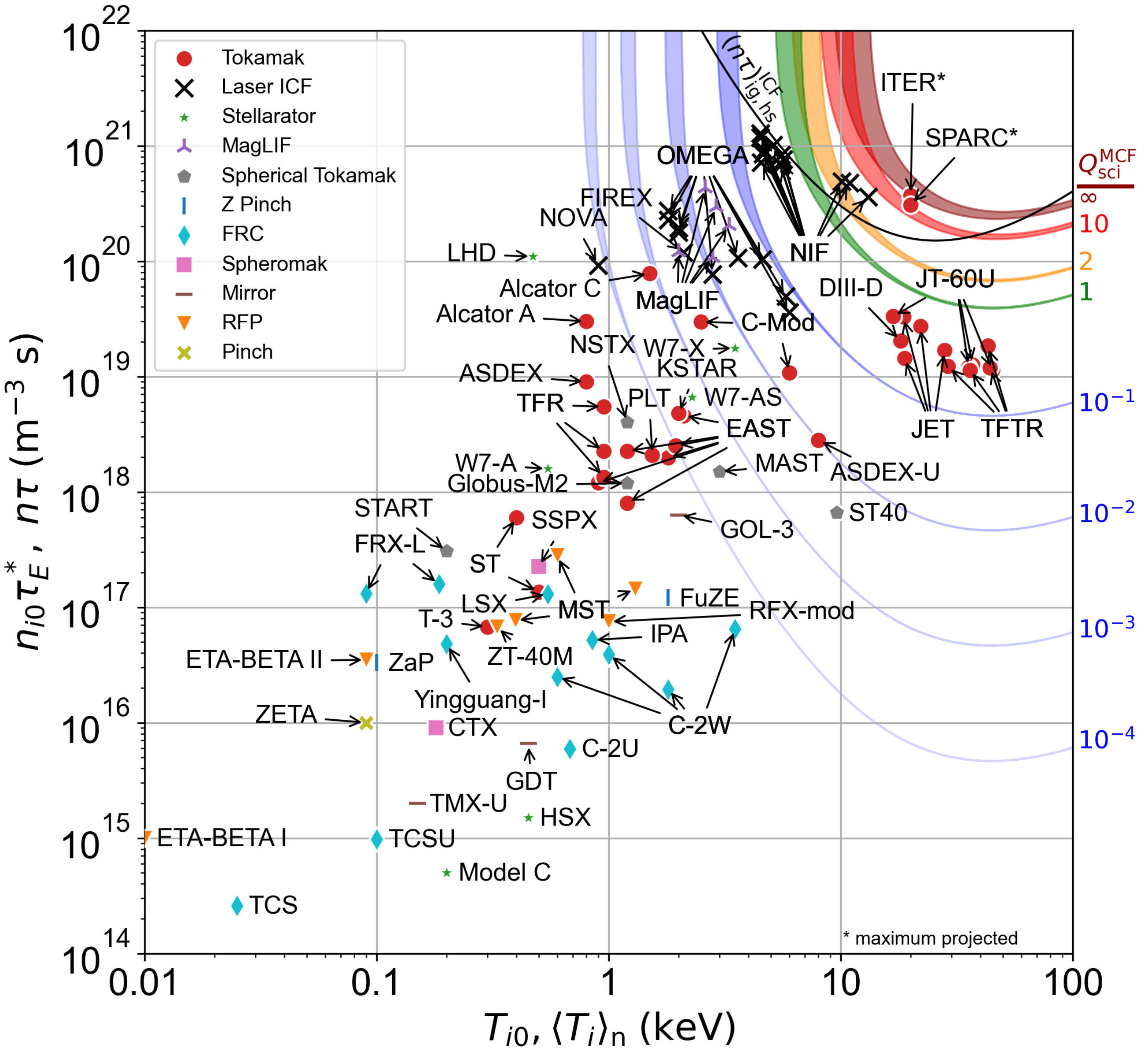


Values of η_{abs} and η_{hs} correspond to NIF shot N191007
A. B. Zylstra et al., *Phys. Rev. Lett.* **126**, 025001 (2021)

Progress towards energy gain

Additional effects:

- impurities
- profile effects
- + more



S.E. Wurzel and S. C Hsu
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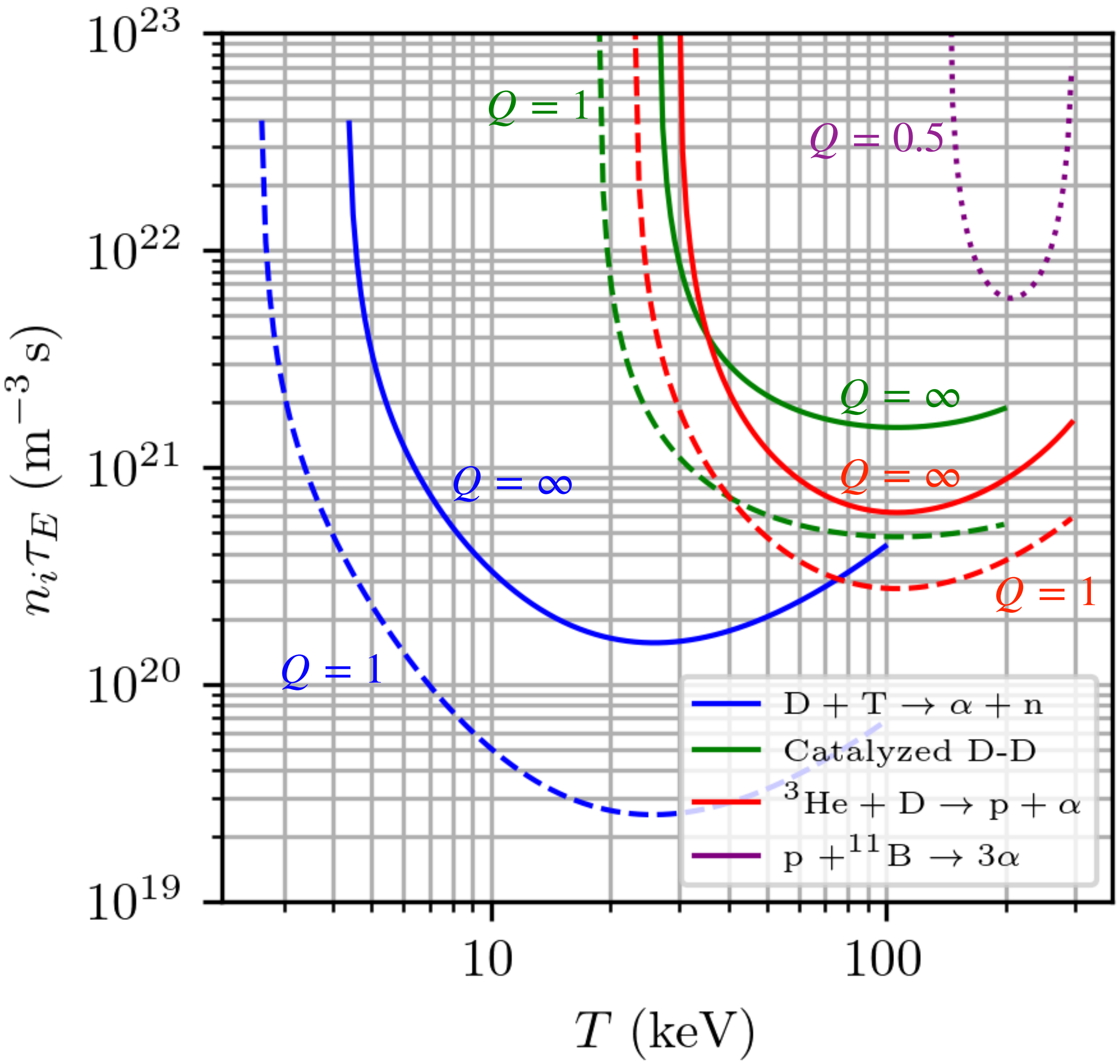
ADVANCED FUELS

Advanced fuels summary

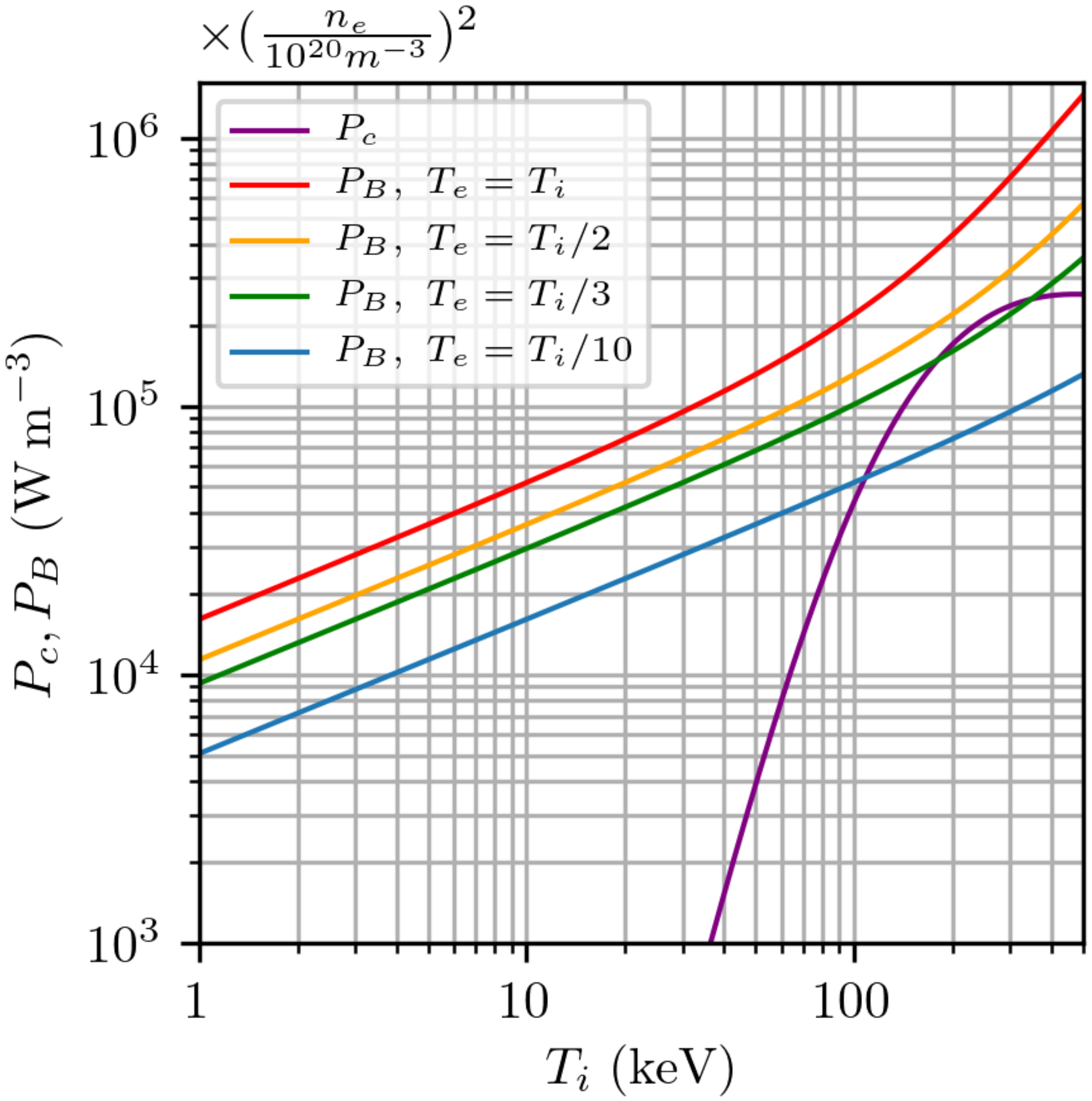
Advanced fuels require significantly higher temperatures and Lawson parameters than D-T



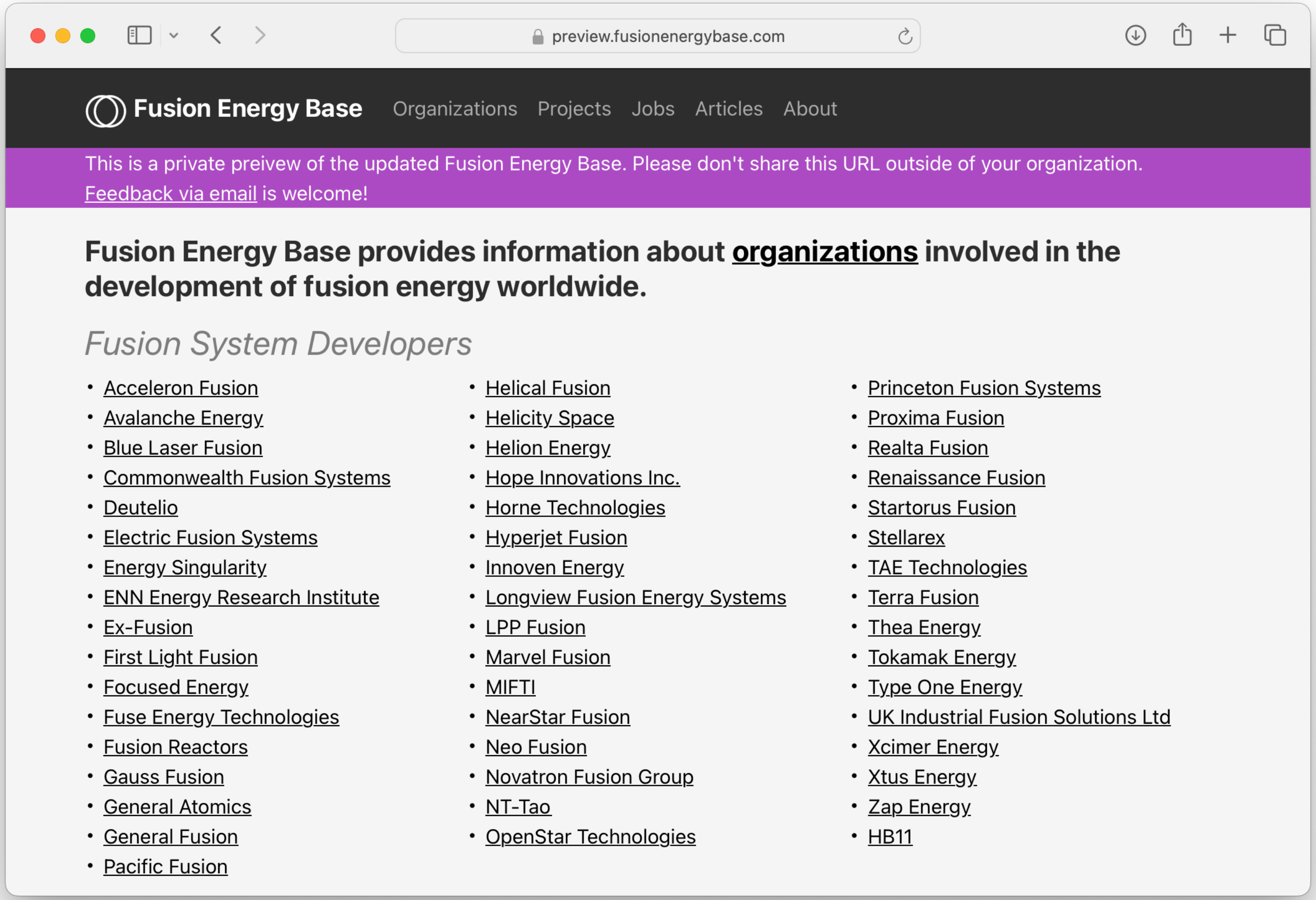
Advanced fuel challenge: p-^{11}B

$$\mathrm{p} + {}^{11}\mathrm{B} \rightarrow 3\alpha \ (8.7 \ \mathrm{MeV})$$

Bremsstrahlung is huge challenge!



Fusion Energy Base (sneak peek)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"Some criteria for a useful thermonuclear reactor," J. D. Lawson, Technical Report No. GP/R 1807 (1955).

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THANKS!

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"Progress toward fusion energy breakeven and gain as measured against the Lawson criterion," S.E. Wurzel and S. C Hsu, Physics of Plasmas **29**, 062103 (2022)

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